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Title

IMPACT OF RADIALLY NON-SYMMETRIC MULTIPLE STENOSES ON BLOOD FLOW THROUGH AN ARTERY

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Abstract:

The impact of radially non-symmetric multiple stenosis on blood flow through an artery have been analyzed by assuming blood as Power-law fluid model. The equations governing the flow of the proposed model are solved and closed from expressions for the blood flow characteristics, namely, dimensionless resistance to flow and wall shear stress at maximum depression for different points of a single loop stenosis. It has been found that the resistance to flow and wall shear stress decreases as stenosis shape parameter increases while both increases as stenosis size and stenosis length increases. This model is able to predict the main characteristics of the physiological flows and may have some interest in biomedical application.

Keywords: Stenosis, Power-law fluid model, Resistance to flow, Wall shear stress, Radially non-symmetric.

Introduction:

The study of blood flow through mammalian circulatory system has been the subject of scientific research for about a couple of centuries. Like most of the problem of nature and life science, it is complex one due to the complicated structure of blood, the circulatory system and their constituent materials. The experimental studies and the theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes and practical applications. The hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. The intimal thickening of stenotic artery was understood as an early process in the beginning of atherosclerosis. The initiation and development of atherosclerotic plaques is depicted in Fig.(1)a and Fig.(1)b. The blood vessels in Fig.(1)a and Fig.(1)b., that we are talking about are the arteries. They are the structures that carry blood from the heart to all the organs and tissues of the body including brain, kidneys, gut, muscles, and the heart itself. There are some illustrations that will help to understand the process of atherosclerosis (vascular disease) and the kinds of problems that can arise in this condition. In recent years many researchers have investigated the blood flow characteristic through artery in the presence of stenosis. Investigators

[1,2,3,4] have emphasized that the formation of intravascular plaques and the impingement of ligaments and spurs on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease. The fruitful study of [5, 6] has pointed out that the variation of resistance to flow and the wall shear stress with the axial distance are physiologically important quantities. [7,8] have shown theoretical results of for the velocity profiles, pressure drop, wall shearing stress and separation phenomena for special geometries for Newtonian model of blood. In the series of the papers [9, 10, 11, 12] the effects on the cardiovascular system can be understood by studying the blood flow in its vicinity. In these studies the behavior of the blood has been considered as a Newtonian fluid. However, it may be noted that the blood does not behave as a Newtonian fluid under certain conditions. It is generally accepted that the blood, being a suspension of cells, behaves as a non-Newtonian fluid at low shear rate [13]. It has been pointed out by [14] that the flow behaviour of blood in a tube of small diameter (less than 0.2 mm) and at less than 20sec^{-1} shear rate, can be represented by a power-low fluid model. In these discussed models, the investigators have not dealt with the radially non-symmetric stenosis. In this present analysis mathematical model for the blood flow through a radially non-symmetric stenosis has been formulated for improved generalized geometry of multiple stenosis located at equispaced points. For simplicity the graphical analysis is performed for a single loop of stenosis having maximum depression at different points.

Formulation of the problem:

In the present analysis, it is assumed that the stenosis develops in the arterial wall and symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. In such a case the radius of artery, R(z) can be written as: Fig (2)

$$\frac{R(z)}{R_0} = \begin{pmatrix} 1\text{-}A[L_0^{(m-1)}(\alpha z\text{-}kd\text{-}(k\text{-}1)L_0) \\ \qquad\qquad -(\alpha z\text{-}kd\text{-}(k\text{-}1)L_0)^m] \end{pmatrix}; \qquad k(d+L_0)\text{-}L_0 \le \alpha z \le k(d+L_0) \tag{1}$$

$$=1; \qquad\qquad \text{otherwise}$$

where $A = \frac{\delta}{R_0 L_0^m}\frac{m^{m/(m-1)}}{(m-1)}$

(2)

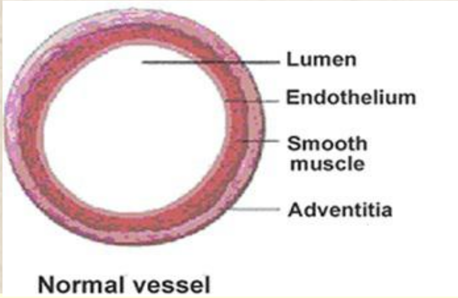

Fig.(1)a

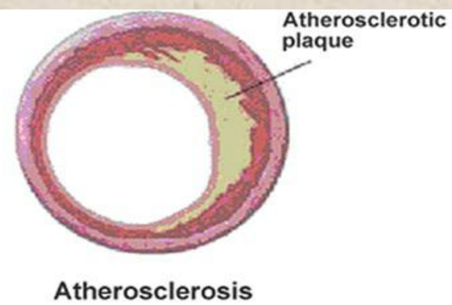

Fig.(1)b

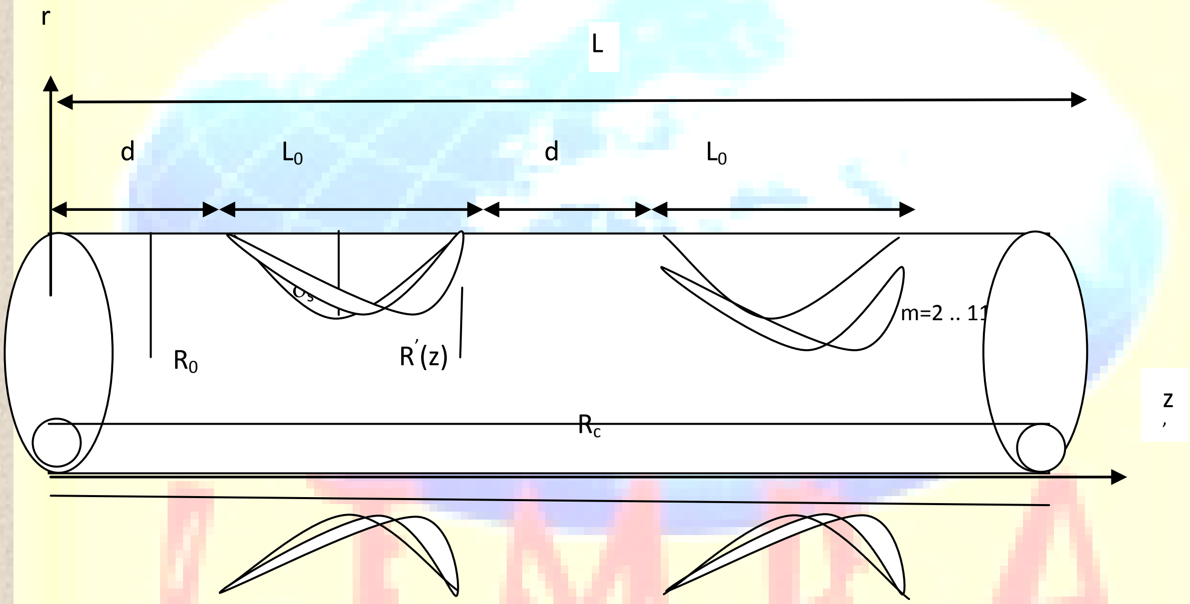

Fig.(2) Geometry of Stenosed artery

R(z) and R_0 is the radius of the artery with and without stenosis, respectively. L is the length of artery and L_0 is the stenosis length, d indicates the distance between equispaced points, k is number of stenosis that appears in arterial lumen, α .is a positive integer ≥ 1 , m is parameter determining the shape of stenosis in artery and δ denotes the maximum height of stenosis at

$$z = \left[\frac{kd \ + \ \ k-1\ L_0 + L_0 / \ m^{1/(m-1)}}{\alpha}\right].$$

Conservation Equation and boundary conditions:

The equation of motion for laminar and incompressible, steady, fully-developed, one-dimensional flow of blood whose viscosity varies along the radial direction in an artery reduces to [4]:

$$\left.\begin{aligned} 0 &= -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\,\tau)}{\partial z}, \\ 0 &= -\frac{\partial P}{\partial r}, \end{aligned}\right\} \qquad (3)$$

where (z, r) are co-ordinates with z measured along the axis and r measured normal to the axis of the artery.

Following boundary conditions are introduced to solve the above equations,

$\partial u / \partial r = 0$ at $r = 0$ $u = 0$ at $r = R(z)$.

τ is finite at $r = 0$ (4)

$P = P_o$ at $z = 0$ $P = P_L$ at $z = L$

Analysis of the problem:

Power-law fluid: Non-Newtonian fluid is that of power-law fluid which have constitutive equation,

$$\left.\begin{aligned} \left(-\frac{du}{dr}\right) &= \left(\frac{\tau}{\mu}\right)^{1/n} = f(\tau), \\ \text{where } \tau &= \left(-\frac{dp}{dz}\right)\frac{R_c}{2} \end{aligned}\right\} \qquad (5)$$

Where u is the axial velocity, □ is the viscosity of fluid, (-dp/dz) is the pressure gradient and n is the flow behaviour index of the fluid.

Solving for u from equation (3), (5) and using the boundary conditions (4), we have,

$$\frac{du}{dr} = \left(\frac{P}{2\mu}\right)^{1/n} [(r - R_C)^{1/n}], \qquad (6)$$

The volumetric flow rate Q can be defined as,

$$Q = \int_0^R 2\pi u r \, dr = \pi \int_0^R r \left(-\frac{du}{dr} \right) dr, \qquad (7)$$

By the help of equations (6) and (7) we have,

$$Q = \left(\frac{P}{2\mu}\right)^{1/n} \left(\frac{n\pi}{(3n+1)} \right) (R)^{[(1/n)+1]} \qquad (8)$$

From equation (8) pressure gradient is written as follows,

$$\frac{dp}{dz} = -2\mu \left(\frac{(3n+1)}{n\pi} Q \right)^n \frac{1}{(R)^{3n+1}} \qquad (9)$$

Integrating equation (9) using the condition P = P_0 at z = 0 and P = P_L at z = L. We have,

$$P_L - P_0 = \left(\frac{(3n+1)}{n\pi} Q \right)^n \frac{2\mu}{R_0^{\,3n+1}} \int_0^L \frac{dz}{{R/R_0}^{1+3n}} \qquad (10)$$

The resistance to flow (resistive impedance) is denoted by λ and defined as follows,

$$\lambda = \frac{P_L - P_0}{Q} \qquad (11)$$

The resistance to flow from equation (11) using equations (10) can write as:

$$\lambda_0 = \left(\frac{(3n+1)Q}{n\pi} \right)^n \frac{2\mu}{Q R_0^{\,3n+1}} \left\{ \int_0^d dz + \int_d^{d+L_0} \frac{dz}{{R/R_0}^{3n+1}} + \int_{d+L_0}^{2d+L_0} dz + \int_{2d+L_0}^{2(d+L_0)} \frac{dz}{{R/R_0}^{3n+1}} + \int_{2(d+L_0)}^{L} dz \right\} \qquad (12)$$

When there is no stenosis in artery then R = R_0, the resistance to flow,

$$\lambda_N = \left(\frac{(3n+1)}{n\pi} Q \right)^n \frac{2\mu}{Q R_0^{\,3n+1}} L \qquad (13)$$

From equation (11) and (12) the ratio of (λ_0 / λ_N) is given as;

$$\lambda=\frac{\lambda_0}{\lambda_N}=1-\frac{K L_0}{\alpha L}+\frac{1}{L}\int_{\frac{d}{\alpha}}^{\left(\frac{d+L_0}{\alpha}\right)}\frac{dz}{\left(1-\frac{\delta}{R_0L_0^m}\frac{m^{m/(m-1)}}{(m-1)}\left[L_0^{(m-1)}(\alpha z-d)-(\alpha z-d)^m\right]\right)^{3n+1}} \qquad (14)$$

Now the ratio of shearing stress at the wall can be written as;

$$\frac{\tau_R}{\tau_N}=\left(\frac{R_0}{R}\right)^{-3n} \qquad (15)$$

$$\tau=\frac{\tau_R}{\tau_N}=\frac{1}{\left(1-\frac{\delta}{R_0}\right)^{3n}} \qquad (16)$$

The apparent viscosity (μo/μ) is defined as follows ;

$$\frac{\mu_0}{\mu}=\frac{1}{(R/R_0)^4 f(\bar{y})} \qquad (17)$$

$$\textit{where} \quad f(\bar{y})=1-\frac{16}{7}\bar{y}^{1/2}+\frac{4}{3}\bar{y}-\frac{1}{21}\bar{y}^4,$$

$$\textit{with} \quad \bar{y}=\frac{R_c}{R}<<1\,.$$

Result and Discussion:

In order to have estimate of the quantitative effects of stenosis shape parameter (m= 2...11), stenosis size, stenosis length on resistance to flow, wall shear stress and apparent viscosity, computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, wall shear stress apparent viscosity for diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Fig 3-6 by using the values of parameter based on experimental data in stenosed artery. Fig.3 reveals the variation of resistance to flow (λ) with stenosis shape parameter (m). It is observed that the resistance to flow (λ) decreases as stenosis shape parameter (m) increases, maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. The result is consisting with the result of [14].

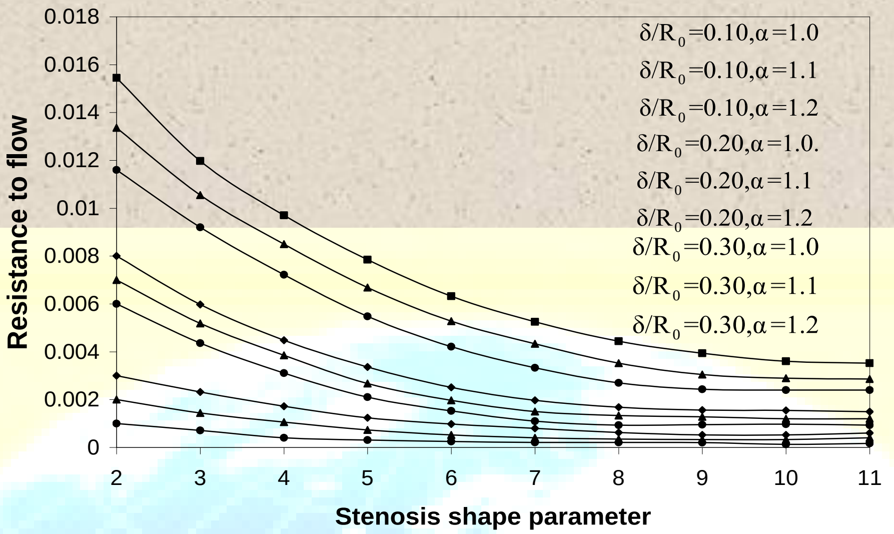

Fig.3 Variatuin of resistance to flow with stenosis shape parameter

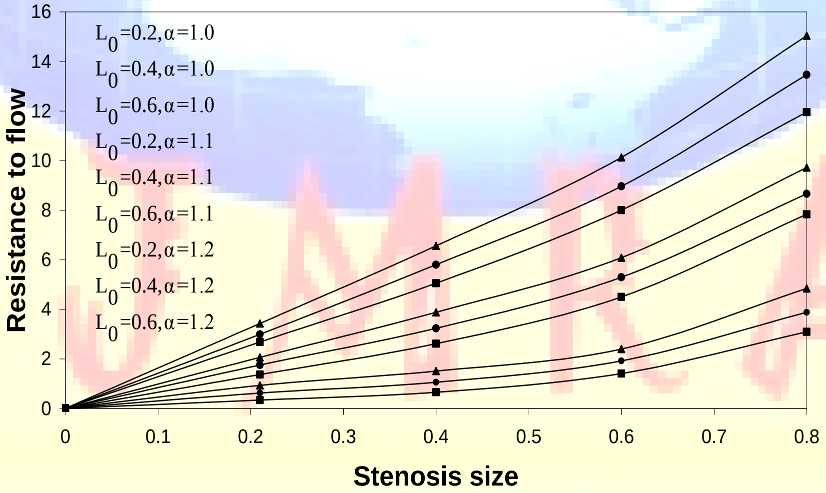

Fig. 4 Variation of resistance to flow with Stenosis size

Fig.4 consists the variation of resistance to flow (λ) with stenosis size (δ/R_0). It is evident that resistance to flow increases as stenosis size increases. Resistance to flow increase as stenosis grows or radius of artery decreases (this referred to as Fahraeus-Lindquist effect in very thin tubes). In Fig.5 the variation of apparent viscosity with stenosis size (m) has been shown. This figure depicts that apparent viscosity increases as stenosis size increases. As the stenosis grows, the apparent viscosity increases in the stenotic region. These results are similar with the results of [14]. Fig.6 describes the variation of wall shear stress (τ) with stenosis size. This figure depicts that wall shear stress (τ) increases as stenosis size increases. These results are consistent to the observation of [12].

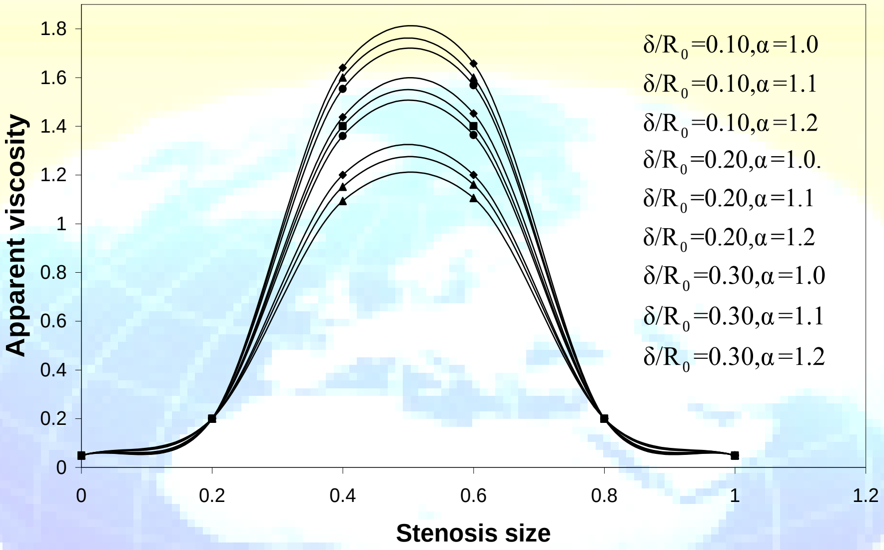

Fig.5 Variation of apparent viscosity with stenosis size

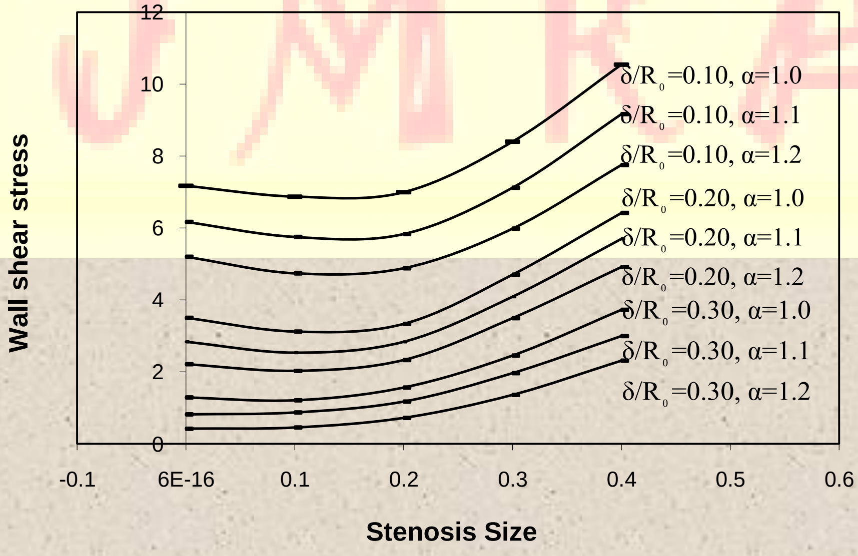

Fig.6 Variation of Wall shear stress with Stenosis Size

CONCLUSION:

In his paper, we have studied the effect of stenosis shape parameter on resistance to blood flow, wall shear stress and apparent viscosity in an artery by introducing blood as Power-law fluid model. It has been concluded that the resistance to blood flow, wall shear stress and apparent viscosity increases as stenosis size and stenosis length increases while decreases as stenosis shape parameter increases. So it has concluded that the results were greatly influenced by the change of stenosis shape parameter. This model is able to predict the main characteristics of the physiological flows and may have some interest in biomedical application.

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